## Official Nomenclature of US and European Societies of Rheology (2018-10)

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Name	Definition	Symbol	SI units <sup>a</sup>
Direction of flow (simple shear)		<i>x</i> <sub>1</sub>	m
Direction of velocity gradient (simple shear)		<i>x</i> <sub>2</sub>	m
Neutral direction (simple shear)		<i>x</i> <sub>3</sub>	m
Shear or normal force	та	F	Ν
Velocity	$dx_1/dt$	$v_1$	$m s^{-1}$
Acceleration	$dv_1/dt$	<i>a</i> <sub>1</sub>	m s <sup>-2</sup>
Shear stress <sup>b</sup>	F/A	σ	Ра
Shear strain	$dx_1/dx_2$	γ	_
Shear rate	$ dv_1/dx_2 $	Ϋ́	s <sup>-1</sup>
Vorticity <sup>c</sup>	$-dv_1/dx_2$	$\omega_3$	$s^{-1}$
Viscosity	$ \sigma /\dot{\gamma}$	$\eta(\dot{\gamma})$	Pa s
Yield stress		$\sigma_y$	Ра
Yield strain		$\gamma_y$	_
First normal stress difference	$\sigma_{11} - \sigma_{22}$	$N_1$	Pa
Second normal stress difference		$N_2$	Ра
First normal stress coefficient	$\frac{\sigma_{22} - \sigma_{33}}{N_1/\dot{\gamma}^2}$	$\Psi_1$	Pa s <sup>2</sup>
Second normal stress coefficient	$N_2/\dot{\gamma}^2$	$\Psi_2$	Pa s <sup>2</sup>
Normal stress ratio	$-N_{1}/N_{2}$	$\Psi(\dot{\gamma})$	_
Zero-shear viscosity	$\eta(\dot{\gamma} \to 0)$	$\eta_0$	Pa s
(limiting low shear rate viscosity)			
Zero-shear first normal stress coefficient	$\frac{\Psi_1(\dot{\gamma} \to 0)}{\eta = K \dot{\gamma}^{n-1}}$	$\Psi_{1.0}$	Pa s <sup>2</sup>
Consistency index in power law for viscosity	$\eta = K \dot{\gamma}^{n-1}$	n	_
Constant in power law for viscosity	see above equation	K	Pa s <sup>n</sup>

### TABLE I. Steady simple shear (viscometric flow)

<sup>a</sup> SI allows either a dot between units or a space, as used here.

<sup>b</sup> A is the area in m<sup>2</sup>.

<sup>c</sup> In the fluid dynamics community a prefactor of 1/2 is used in the definition of the vorticity.

Name	Definition	Symbol	Units
Simple shear			
Shear modulus of a solid	$\sigma/\gamma$	G	Pa
Relaxation modulus	$\sigma(t)/\gamma$	G(t)	Pa
Relaxation strength in discrete spectrum	_	$g_i$	Pa
Relaxation time in discrete spectrum	_	$\tau_i$	S
Continuous relaxation spectrum	_ <sup>a</sup>	$H(\tau)$	Pa
Orthogonal superposition complex modulus <sup>b,c</sup>	_	$G_{\perp}^{*}(\omega,\dot{\gamma})$	Pa
Parallel superposition complex modulus <sup>b,c</sup>	-	$G_{\parallel}^{*}(\omega,\dot{\gamma})$	Pa
Memory function	-dG(s)/ds	m(s)	Pa s <sup>-1</sup>
Creep compliance (shear)	$\gamma(t)/\sigma$	J(t)	Pa <sup>-1</sup>
Equilibrium compliance of solid	$J(t) \ (t \to \infty)$	J <sub>e</sub>	Pa <sup>-1</sup>
Recoverable compliance	$J(t) - t/\eta_0$	$J_r(t)$	Pa <sup>-1</sup>
Steady-state compliance of fluid	$J(t) - t/\eta_0 (t \to \infty)$	$J_s^0$	Pa <sup>-1</sup>
Continuous retardation spectrum <sup>d</sup>	d	$L(\tau)$	Pa <sup>-1</sup>
Small-amplitude oscillatory shear	·	•	
Strain amplitude	$\gamma(t) = \gamma_0 \sin \omega t$	γο	_
Loss angle (phase angle)	$\sigma(t) = \sigma_0 \sin(\omega t + \delta)$	δ	rad
Stress amplitude	$\sigma(t) = \sigma_0 \sin(\omega t + \delta)$	$\sigma_0$	Pa
Complex modulus	G' + iG''	G*	Pa
Absolute magnitude of <i>G</i> <sup>*</sup>	$\sigma_0/\gamma_0$	<i>G</i> *  or	Ра
		$G_d$	
Storage modulus	$G_d \cos \delta$	<i>G'</i>	Pa
Loss modulus	$G_d \sin \delta$	<i>G''</i>	Pa
Complex viscosity	$\eta' - i\eta''$	$\eta^*$	Pa s
Absolute magnitude of $\eta^*$	$\sigma_0/\omega\gamma_0$	$ \eta^* $	Pa s
Dynamic viscosity (in phase with strain rate)	$G''/\omega$	$\eta'$	Pa s
Out-of-phase (with strain rate) component of $\eta^*$	G'/w	$\eta^{\prime\prime}$	Pa s
Complex compliance	J' - iJ''	$J^*$	Pa <sup>-1</sup>
Absolute magnitude of $J^*$	$\gamma_0/\sigma_0 = 1/G_d$	J*	Pa <sup>-1</sup>
Storage compliance	$(\gamma_0/\sigma_0)\cos\delta$	J'	Pa <sup>-1</sup>
Loss compliance	$(\gamma_0/\sigma_0)\sin\delta$	J''	Pa <sup>-1</sup>
Plateau modulus <sup>e</sup>	- <sup>e</sup>	$G_N^0$	Pa
Tensile (Uniaxial) Extension °			
Net tensile stress	$\sigma_{zz} - \sigma_{rr}$	$\sigma_E$	Pa
Hencky strain	$\ln(L/L_0)$	8	_
Hencky strain rate	$d(\ln L)/dt$	Ė	$s^{-1}$
Tensile relaxation modulus	$\sigma_E(t)/\varepsilon_0$	E(t)	Pa
Tensile creep compliance	$\overline{\varepsilon_0(t)}/\sigma_E$	D(t)	Pa <sup>-1</sup>

# TABLE II. Linear viscoelasticity

<sup>a</sup>  $G(t) = \int_{-\infty}^{\infty} H(\tau) [\exp(-t/\tau)] d(\ln \tau)$ 

<sup>b</sup> The same subscripts apply to definitions of storage and loss moduli and viscosities.

<sup>c</sup> Also applies to nonlinear phenomena in uniaxial extension.

<sup>d</sup>  $J(t) = \int_{-\infty}^{\infty} L(\tau) [1 - \exp(t/\tau)] d(\ln \tau)$ 

<sup>e</sup> Because there is not a true plateau in G(t) or  $G'(\omega)$ ,  $G_N^0$  is inferred from  $G'(\omega)$  and  $G''(\omega)$  using methods reviewed by Liu *et al.* [Polymer **47**, 4461–4479 (2006)].

Name	Definition	Symbol	SI Units
Vertical shift factor <sup>a</sup>	$b_T G'(T, \omega a_T) = G'(T_0, \omega)$	$b_T$	_
Horizontal shift factor	$b_T G'(T, \omega a_T) = G'(T_0, \omega)$	$a_T$	_
First WLF coefficient	$-c_1(T-T_0)$	<i>c</i> <sub>1</sub>	-
	$\log_{10} a_T = \frac{1}{[c_2 + (T - T_0)]}$		
Second WLF coefficient	$-c_1(T-T_0)$	<i>C</i> <sub>2</sub>	Κ
	$\log_{10} a_T = \frac{-c_1(T - T_0)}{[c_2 + (T - T_0)]}$		
Activation energy for flow	$a_T = \exp\left[\frac{E_a}{R}\left(\frac{1}{T} - \frac{1}{T}\right)\right]$	$E_a$	kJ mol <sup>-1</sup>
	$u_T = \exp\left[\frac{1}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$		

TABLE III. Shift factors for time-temperature superposition

<sup>a</sup> For rubbers,  $b_{\rm T}$  is given by  $T_0\rho_0/T\rho$ , where  $\rho$  is mass density, with  $\rho_0 = \rho(T_0)$  [J. D. Ferry, Viscoelastic properties of polymers, 3<sup>rd</sup> Ed., Wiley, NY, 1980]. It has also been found to be appropriate for polymers in general.

Name	Definition	Symbol	SI Units	
Stress relaxation (step strain)				
Strain amplitude		$\gamma_0$	_	
Relaxation modulus (nonlinear)	$\sigma(t)/\gamma_0$	$G(t, \gamma_0)$	Ра	
Damping function in shear	$G(t,\gamma_0)/G(t)$	$h(\gamma_0)$	_	
First normal stress relaxation function		$N_1(t,\gamma_0)$	Pa	
Second normal stress relaxation function		$N_2(t,\gamma_0)$	Ра	
First normal stress relaxation coefficient	$N_1(t,\gamma_0)/\gamma_0^2$	$\Psi_1(t,\gamma_0)$	Ра	
Second normal stress relaxation coefficient	$N_2(t,\gamma_0)/\gamma_0^2$	$\Psi_2(t,\gamma_0)$	Pa	
Start-up of steady shear (at fixed shear rate	)			
Shear stress growth function		$\sigma^+(t,\dot{\gamma})$	Ра	
Shear stress growth coefficient	$\sigma^+(t,\dot{\gamma})/\dot{\gamma}$	$\eta^+(t,\dot{\gamma})$	Pa s	
First normal stress growth function	$\sigma_{11} - \sigma_{22}$	$N_1^+(t,\dot{\gamma})$	Ра	
First normal stress growth coefficient	$N_1^+(t,\dot{\gamma})/\dot{\gamma}^2$	$\Psi_1^+(t,\dot{\gamma})$	Pa s <sup>2</sup>	
Second normal stress growth function	$\sigma_{22} - \sigma_{33}$	$N_2^+(t,\dot{\gamma})$	Ра	
Second normal stress growth coefficient	$N_2^+(t,\dot{\gamma})/\dot{\gamma}^2$	$\Psi_2^+(t,\dot{\gamma})$	Pa s <sup>2</sup>	
Stress Ratio	$N_1(\dot{\gamma})/\sigma(\dot{\gamma})$	SR	_	
Cessation of steady shear ( $\dot{\gamma} = 0$ from $t = 0$	))			
Shear stress decay function		$\sigma^{-}(t,\dot{\gamma})$	Ра	
Shear stress decay coefficient	$\sigma^{-}(t,\dot{\gamma})/\dot{\gamma}$	$\eta^{-}(t,\dot{\gamma})$	Pa s	
First normal stress decay function	$\sigma_{11} - \sigma_{22}$	$N_1^-(t,\dot{\gamma})$	Pa	
First normal stress decay coefficient	$N_1^-(t,\dot{\gamma})/\dot{\gamma}^2$	$\Psi_1^-(t,\dot{\gamma})$	Pa s <sup>2</sup>	
Second normal stress decay function	$\sigma_{22} - \sigma_{33}$	$N_2^-(t,\dot{\gamma})$	Ра	
Second normal stress decay coefficient	$\sigma_{22} - \sigma_{33} = N_2^-(t,\dot{\gamma})/\dot{\gamma}^2$	$\Psi_2^-(t,\dot{\gamma})$	Pa s <sup>2</sup>	
Creep and creep recovery (recoil)				
Creep compliance	$\gamma(t,\sigma)/\sigma$	$J(t,\sigma)$	Pa <sup>-1</sup>	
Steady-state compliance <sup>a</sup>	$J(t \to \infty, \sigma)$	$J_s(\sigma)$	Pa <sup>-1</sup>	
Recoverable strain	$\gamma[t_0,\sigma] - \gamma[t,\sigma]$	$\gamma_r(t',\sigma)$	_	
(after $t_0$ when $\sigma \to 0$ )	$t > t_0$	$t' = t - t_0$		
Ultimate recoil	$\gamma_r(t' \to \infty, \sigma)$	$\gamma_{\infty}(\sigma)$	_	
Steady-state recoverable compliance <sup>a</sup>	$\gamma_{\infty}(\sigma)/\sigma$	$J_r(\sigma)$	Pa <sup>-1</sup>	

# TABLE IV. Nonlinear viscoelasticity in shear

<sup>a</sup> Although measured in different ways, the steady-state compliance and the steady-state recoverable compliance should be equal to each other according to the Boltzmann principle.

Name	Definition	Symbol	SI Units
Tensile (uniaxial) extension			
Engineering strain <sup>a</sup>	$(L - L_0)/L_0$	ε	_
Engineering stress <sup>a</sup>	$F/A_0$	σ	Pa
Young's modulus of a solid	σ/ε	Ε	Pa
Net tensile stress (true)	$\sigma_{zz} - \sigma_{rr}$	$\sigma_{\varepsilon}$	Pa
Hencky strain	$\ln(L/L_0)$	$\varepsilon$ or $\epsilon$	_
Hencky strain rate	$d(\ln L)/dt$	ė́ or ė́	$s^{-1}$
Tensile stress growth function		$\sigma_E^+(t,\dot{\varepsilon})$	Pa
Tensile stress growth coefficient	$\sigma_E^+(t,\dot{\varepsilon})/\dot{\varepsilon}$	$\eta_E^+(t,\dot{\varepsilon})$	Pa s
Extensional viscosity	$\eta_E^+(t,\dot{\varepsilon}) \ t \to \infty$	$\eta_E(\dot{\varepsilon})$	Pa s
Tensile creep compliance	$\varepsilon(t)/\sigma_E$	$D(t,\sigma_E)$	Pa <sup>-1</sup>
Recoverable strain	$\varepsilon[t_0, \sigma_E] - \varepsilon[t, \sigma_E]$	$\varepsilon_r(t',\varepsilon)$	-
(after $t_0$ where $\sigma_{\rm E} \rightarrow 0$ )	$t > t_0$	$t' = t - t_0$	
Biaxial extension (symmetrical)			
Biaxial strain	$\ln(R/R_0)$	$\varepsilon_B$	_
Biaxial strain rate	$d(\ln R)/dt$	$\dot{\varepsilon}_B$	s <sup>-1</sup>
Net stretching stress	$\sigma_{rr} - \sigma_{zz}$	$\sigma_B$	Pa
Biaxial stretch growth function		$\sigma_B^+(t,\dot{\varepsilon}_B)$	Pa
Biaxial stretch growth coefficient	$\sigma_B^+(t,\dot{\varepsilon}_B)/\dot{\varepsilon}_B$	$\eta_B^+(t,\dot{\varepsilon}_B)$	Pa s
Biaxial stress decay coefficient	$\sigma_B^-(t,\dot{\varepsilon}_B)/\dot{\varepsilon}_B$	$\eta_B^-(t,\dot{\varepsilon}_B)$	Pa s
Biaxial extensional viscosity	$\sigma_B^+(t \to \infty, \dot{\varepsilon}_B)/\dot{\varepsilon}_B$	$\eta_B(\dot{\varepsilon}_B)$	Pa s
Biaxial creep compliance	$\varepsilon_B(t)/\sigma_B$	$D(t,\sigma_B)$	Pa <sup>-1</sup>

TABLE V. Nonlinear viscoelasticity in extension	TABLE V.	Nonlinear	viscoelasticity	in	extension.
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<sup>a</sup> In the mechanics literature, the same symbols are often used for both engineering and true stress and strain, but they are only equivalent in the limit of very small deformations.

### TABLE VI. Rheometry

Name	Definition	Symbol	SI Units
Capillary rheometers			
Apparent wall shear stress <sup>a</sup>	$P_d R/2L$	$\sigma_A$	Pa
Apparent wall shear rate	$4Q/\pi R^3$	Ϋ́A	$s^{-1}$
Wall shear stress	$-\sigma_{rz} (r = R)$	$\sigma_W$	Pa
Wall shear rate	$-dv_z/dr (r = R)$	Ϋ́W	$s^{-1}$
Cone-plate rheometers			
Cone angle	Figure 1	β	rad
Angular rotation	Figure 1	$\varphi$	rad
Angular velocity	dφ∕dt	Ω	rad s <sup>-1</sup>
Torque <sup>b</sup>	$2\pi R^3 \sigma_{\varphi\theta}/3$	М	N m
Normal thrust	Figure 1	$F_z$	Ν

<sup>a</sup>  $P_d$  = driving (reservoir) pressure (exit pressure neglected)

<sup>b</sup> Approximation valid for  $\beta < 0.1$  rad.

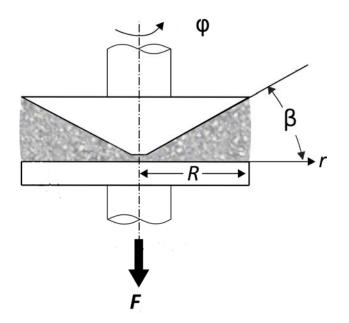


Figure 1. Symbols describing cone-plate geometry.

### **TABLE VII.** Solutions

Name	Definition	Symbol	SI units
Concentration		С	kg m <sup>-3 a</sup>
Overlap		<i>C</i> *	kg m <sup>3 a</sup>
concentration			
Solvent viscosity		$\eta_s$	Pa s
Relative viscosity	$(\eta/\eta_s)$	$\eta_r$	_
Specific viscosity	$(\eta_r - 1)$	$\eta_{sp}$	-
Reduced viscosity	$\eta_{sp}/c$	$\eta_{red}$	$m^3 kg^{-1 b}$
Intrinsic viscosity	$\lim(\eta_{\rm red}, \dot{\gamma} \to 0, c \to 0)$	$[\eta]$	$m^3 kg^{-1 b}$
Viscosity of matrix <sup>c</sup>		$\eta_m$	Pa s
Solvent contribution		$\sigma^{s}$	Pa
to the stress tensor			
Polymeric		$\sigma^p$	Pa
contribution to the			
stress tensor			

<sup>a</sup> Units of g mL<sup>-1</sup> are often used.

<sup>b</sup> Units of mL g<sup>-1</sup> are often used.

<sup>c</sup> Often used for nanocomposites, i.e., particles in a viscoelastic matrix.

# TABLE VIII. Suspensions

Name	Definition	Symb	SI
		oľ	Units
Volume fraction solid	$V_{solid}/V_{suspension}$	$\phi$	-
Maximum packing fraction		$\phi_{max}$	_
Suspending medium viscosity <sup>a</sup>		$\eta_m$	Pa s
Effective viscosity of suspension	$\Sigma_{12}/\dot{\gamma}$	η	Pa s
Relative viscosity of suspension	$\eta/\eta_f$	$\eta_r$	_
Particle contribution to $\eta$		$\eta_p$	Pa s
Local stress tensor		$\boldsymbol{\sigma}(\boldsymbol{x},t)$	Pa
Total bulk stress	$\Sigma^f + \Sigma^p$	Σ	Pa
Fluid (suspending medium) contribution		$\mathbf{\Sigma}^{f}$	Pa
Particle contribution		$\mathbf{\Sigma}^p$	Pa
First normal stress difference	$\Sigma_{11} - \Sigma_{22}$	$N_1$	Pa
Second normal stress difference	$\frac{\Sigma_{11} - \Sigma_{22}}{\Sigma_{22} - \Sigma_{33}}$	$N_2$	Pa
Dimensionless first normal stress difference	$-N_1/\eta_f \dot{\gamma}$	$\gamma_1$	_
Dimensionless second normal stress	$-N_2/\eta_f \dot{\gamma}$	$\gamma_2$	_
difference			
Particle pressure	$-\frac{1}{3} \left( \Sigma_{11}^{p} + \Sigma_{22}^{p} + \Sigma_{33}^{p} \right)$	П	Pa
Hydrodynamic particle stress		$\Sigma^{H}$	Ра
Interparticle stress		$\Sigma^{IP}$	Pa
Brownian stress		$\Sigma^B$	Pa

<sup>a</sup> Sometimes the subscript f is used to indicate the fluid viscosity, but it seems more adequate to use m which stands for "medium" if the suspending fluid is itself rheologically complex or "matrix" in nanocomposites. Also,  $\mu$  is often used to denote fluid viscosity.

# TABLE IX. Interfacial and surface rheology

Name	Definition	Symbol	SI Units
Interfacial or surface tension <sup>a</sup>		$\sigma_{lpha,eta}$	Pa m
Surface pressure	$\sigma^0_{\alpha,\beta} - \sigma_{\alpha,\beta}(F^s/A)$	Π	Pa m
Interfacial shear stress	$F^{s}/L$	$\sigma^s$	Pa m
Interfacial shear strain	$dx_1/dx_2$	$\gamma^{s}$	_
Interfacial shear rate	$dv_1/dx_2$	$\dot{\gamma}^{s}$	$s^{-1}$
Interfacial dilatational strain	$\ln(A/A_0)$	$\alpha^{s}$	—
Interfacial dilatational strain rate	$d(\ln A)/dt$	ά <sup>s</sup>	s <sup>-1</sup>
Interfacial concentration	_	Г	kg m <sup>-2</sup>
Steady shear and dilatation			
Interfacial shear viscosity	$\sigma^s/\dot{\gamma}^s$	$\eta^s$	Pa s m
Interfacial dilatational viscosity	$\sigma^s/\dot{\alpha}^s$	κ <sup>s</sup>	Pa s m
Simple shear			
Interfacial shear modulus	$\sigma^s/\gamma^s$	$G^{s}$	Pa m
Relaxation modulus (shear)	$\sigma^{s}(t)/\gamma^{s}$	$G^{s}(t)$	Pa m

Pure dilatation	545	175	D
Interfacial dilatational modulus	$\sigma^s/\alpha^s$	$K^s$	Pa m
Dilatational storage modulus	$\sigma^{s}(t)/\alpha^{s}$	$K^{s}(t)$	Pa m
Gibbs elasticity (surfactants) <sup>b</sup>	$d\sigma_{\alpha,\beta}/d(\ln A)$	$K_{\Pi}$	Pa m
Small-amplitude oscillatory shear			
Strain amplitude	$\gamma^s = \gamma_0^s \sin(\omega t)$	$\gamma_0^s$	—
Phase angle (loss angle)	$\sigma^{s}(t) = \sigma_{0}^{s} \sin(\omega t + \delta)$	δ	rad
Stress amplitude	$F^{s}/L$	$\sigma_0^s$	Pa m
Complex interfacial shear modulus	$G^{s'} + iG^{s''}$	G <sup>s</sup> *	Pa m
Absolute magnitude of <i>G</i> <sup>s*</sup>	$\sigma_0^s/\gamma_0^s$	$ G^{s*} $	Pa m
Storage modulus	$ G^{s*} \cos\delta$	$G^{s'}$	Pa m
Loss modulus	$ G^{s*}  \sin \delta$	G <sup>s</sup> "	Pa m
Complex shear viscosity	$\eta^{s'} - i\eta^{s''}$	$\eta^{s*}$	Pa s m
Absolute magnitude of $\eta^{s*}$	$\sigma_0^s / \omega \gamma_0^s$	$ \eta^{s*} $	Pa s m
Dynamic shear viscosity (in phase with strain	$G^{s''}/\omega$	$\eta^{s'}$	Pa s m
rate)			
Out-of-phase (with strain rate) component of	$G^{s'}/\omega$	$\eta^{s''}$	Pa s m
$\eta^{s*}$			
Small-amplitude oscillatory dilatation			
Dilatational strain amplitude	$\alpha^s = \alpha_0^s \sin(\omega t)$	$\alpha_0^s$	_
Complex dilatational modulus <sup>b</sup>	$K^{s'} + iK^{s''}$	$K^{s*}$	Pa m
Absolute magnitude of $K^{s*b}$	$\sigma_0^s/\alpha_0^s$	$ K^{s*} $	Pa m
Storage dilatational modulus <sup>b</sup>	K <sup>s</sup> '	K <sup>s</sup> '	Pa m
Loss dilatational modulus <sup>b</sup>	<i>K<sup>s</sup>''</i>	K <sup>s</sup> "	Pa m
Complex dilatational viscosity	$\kappa^{s'} + i\kappa^{s''}$	$\kappa^{s*}$	Pa s m
Absolute magnitude of $\kappa^{s*}$	$\sigma_0^s/\omega\delta_0^s$	$ \kappa^{S*} $	Pa s m
Dynamic dilatational viscosity	$K^{s''}/\omega$	$\kappa^{s'}$	Pa s m
Out-of-phase component of $\kappa^{s*}$	$K^{s'}/\omega$	$\kappa^{s''}$	Pa s m
Other properties			
Creep compliance (shear)	$\gamma^{s}(t)/\sigma^{s}$	$J^{s}(t)$	$Pa^{-1} m^{-1}$
Equilibrium compliance of solid	$J^{s}(t) (t \to \infty)$	$J_e^s$	$Pa^{-1} m^{-1}$
Recoverable compliance	$J^{s}(t) - t/\eta_{0}^{s}$	$J_r^s(t)$	Pa <sup>-1</sup> m <sup>-1</sup>
Steady-state compliance of fluid	$J^{s}(t) - t/\eta_{0}^{s} \ (t \to \infty)$	$J_0^s$	Pa <sup>-1</sup> m <sup>-1</sup>
Extensional viscosity	$\eta_E^s(t,\dot{\varepsilon}^s) \ (t \to \infty)$	$\eta_E^s(\dot{\varepsilon}^s)$	Pa s m

<sup>a</sup> In some cases the symbols  $\alpha$  or  $\Gamma$  are used to denote surface tension (not to be confused with interfacial dilatational strain or surface concentration, respectively).

<sup>b</sup> Sometimes the symbol E is used instead of K, but this should not be confused with Young's modulus. The use of K for a compression modulus is recommended.

### TABLE X. Molecular description of entangled polymers

- atube diameter (typically in nm); average entanglement spacing / mesh size<br/> $\sqrt{\langle R^2 \rangle_0 M_e / M}$ b\_KKuhn segment length aftension in a chain segment (typically in N)
- $f_{\rm max}$  maximum tension in a chain segment
- $k_{\rm B}$  Boltzmann's constant,  $1.38 \times 10^{-23}$  J K<sup>-1</sup>
- *L* mean tube contour length
- M molecular weight <sup>b</sup>, kg mol<sup>-1</sup>
- $M_{\rm C}$  critical molecular weight for effect of entanglements on  $\eta_0$ , kg mol<sup>-1</sup>
- $M'_{\rm C}$  molecular weight for effect of entanglements on  $J_{\rm S}^0$ , kg mol<sup>-1</sup>
- $M_{\rm e}$  molecular weight between entanglements <sup>c</sup> ( $\rho RT/G_N^0$ )
- $N_{\rm K}$  number of Kuhn segments in equivalent freely jointed chain <sup>a</sup>
- p packing length <sup>d</sup>  $(M/[\langle R^2 \rangle_0 \rho N_A])$
- $p^2$  parameter to denote degree of branch point hopping within tube (fraction of tube diameter)
- *R* end-to-end distance of polymer molecule
- $R_{\rm max}$  fully extended chain length
- *s* tube contour variable (curvilinear coordinate along tube)
- **S** tube orientation tensor
- Z number of entanglement segments per molecule  $^{\circ}$  ( $M/M_{e}$ )

#### Greek letters

- $\alpha$  dilution exponent for  $M_{\rm e}$
- $\zeta$  friction coefficient
- $\zeta_0$  monomer friction coefficient
- $\lambda$  chain stretch; stretch ratio
- $\xi$  correlation length; characteristic size scale (blob)
- $\tau_{\rm d}$  reptation (tube disengagement) time
- $\tau_{\rm e}$  Rouse time of an entanglement strand  $(\tau_{\rm R}/3\pi^2 Z^2)$
- $\tau_{\rm p}$  relaxation time of the  $p^{\rm th}$  mode (p is the mode index)
- $\tau_{\rm r}$  Rouse reorientation relaxation time
- $\tau_{\rm R}$  Rouse stress relaxation time  $[(\zeta N^2 b^2)/(6\pi^2 k_{\rm B}T)]$
- $\tau_{\rm s}$  stretch relaxation time <sup>f</sup>

<sup>a</sup>  $b_{\rm K}$  and  $N_{\rm K}$  are defined by the following relationships:  $\langle R^2 \rangle = b_{\rm K}^2 N_{\rm K}$ ;  $R_{max} = b_{\rm K} N_{\rm K}$ 

<sup>b</sup> IUPAC recommends *molar mass* (MM), which has SI units of g mol<sup>-1</sup>. But *molecular weight* (MW) is widely used, and ACS accepts both terms. However, MW is formally a dimensionless ratio that is numerically very close to MM (g mol<sup>-1</sup>), and one cannot "change its units". The number often called "molecular weight (kg mol<sup>-1</sup>)" is formally MW/1000 (no units). This quantity should more properly be called *molar mass* with units of kg mol<sup>-1</sup>. But it is so widely called molecular weight that it seems hopeless to try to change it now.

<sup>c</sup> This is the definition originally proposed by John Ferry. The following alternative definition was introduced much later in the context of Doi-Edwards model. It should be used in the context of the development of, or comparisons with tube-model theories:  $M_e = \frac{4}{\tau} \rho RT / G_N^0$ .

<sup>d</sup> For a discussion of *p* see Fetters *et al*. [Macromolecules **27**, 4639 (1994)].

<sup>e</sup> In tube-model theory, Z is called "number of tube segments" and is defined based on the  $M_e$  of footnote (c), i.e., using the Doi-Edwards  $\frac{4}{5}$  factor. Hence, when comparing with or using tube models, the  $\frac{4}{5}$  factor should be used.

<sup>f</sup> The Rouse reorientation relaxation time was called the rotational relaxation time by Doi and Edwards and is conjectured to be equal to  $2\tau_R$ .

### Table XI. Stress and strain tensors

Total stress tensor	σ
Extra stress tensor	τ
Strain tensor for linear viscoelasticity	γ
Cauchy tensor	С
Finger tensor	<b>B</b> or <b>C</b> <sup>-1</sup>
Doi-Edwards strain tensor	Q
Rate-of-strain tensor <sup>a</sup>	$\dot{\boldsymbol{\gamma}} = \boldsymbol{\nabla} \mathbf{v} + (\boldsymbol{\nabla} \mathbf{v})^T$
Vorticity tensor	$\mathbf{\Omega} = \mathbf{\nabla} \mathbf{v} - (\mathbf{\nabla} \mathbf{v})^T$

<sup>a</sup> An alternative definition, equal to  $\frac{1}{2}\dot{\gamma}$ , is widely used in fluid mechanics and is acceptable, but the symbol **D** should be used for this tensor to avoid confusion:  $\mathbf{D} \equiv \frac{1}{2} [\nabla \mathbf{v} + (\nabla \mathbf{v})^T]$ , and in the fluid dynamics community a factor of  $\frac{1}{2}$  is also used as a prefactor in the definition of the vorticity tensor  $\mathbf{\Omega}$ .

#### Table XII. Dimensionless groups used to describe experimental regimes

Bingham Number <sup>a</sup>	$Bn \equiv (\text{yield stress}) / (\text{shear stress})$
Boussinesq Number	$Bq \equiv (\text{surface shear stress}) / [(\text{bulk subphase shear stress}) \times (\text{perimeter length along which the surface shear stress acts})]$
Capillary number	$Ca = \eta v / \sigma_{\alpha,\beta} \equiv (viscous \text{ forces}) / (surface \text{ forces})$
	or $Ca = \eta \dot{\gamma} R / \sigma_{\alpha,\beta}$
Deborah Number <sup>b,c</sup>	De = (characteristic time of fluid) / (duration of deformation)
	$[e.g., De = \tau \omega]$
Péclet Number	$P\dot{e} = \dot{\gamma}a^2/D_o \equiv (\text{rate of advection}) / (\text{rate of diffusion})$
	$[a = \text{particle radius}, D_o = \text{particle diffusion coefficient}]$
Poisson ratio <sup>d</sup>	$v = -d\varepsilon_{\text{transverse}}/d\varepsilon_{\text{axial}} \equiv \text{negative ratio of transverse to axial strain}$
Reynolds number	$Re = \rho v d / \eta \equiv (\text{inertial forces}) / (\text{viscous forces})$
-	[d = characteristic length scale]
Weissenberg Number <sup>b,e,f</sup>	$Wi \equiv$ (characteristic time of fluid) × (rate of deformation)
	$[e.g., Wi = \tau \dot{\varepsilon} \text{ or } \tau \dot{\gamma}]$

<sup>a</sup> Sometimes the wall shear stress is specifically used in the denominator of the definition, or the ratio (viscosity  $\times$  velocity)/length.

<sup>b</sup> The definitions and uses of these groups are explained in detail in "Weissenberg and Deborah Numbers – Their definition and use", *Rheol. Bulletin* (The Society of Rheology), **79**(2) p.14 (2010).

<sup>c</sup> Specific Deborah numbers are often used, depending on the choice of characteristic time. Popular examples are: Deborah number based on Rouse relaxation time  $(De_R)$  and Deborah number based on longest relaxation time  $(De_d)$ .

<sup>d</sup> The Poisson ratio is also defined as function of Young's and shear modulus ( $\nu = E/2G - 1$ ) in three dimensions (and differently in two dimensions).

<sup>e</sup> The Weissenberg number has sometimes been considered to be  $N_1/\sigma$ . However, this is a ratio of dependent rather than independent variables and thus describes data rather than experimental conditions. The quantity  $N_1/\sigma$  is often called the *stress ratio*.

<sup>f</sup> Specific Weissenberg numbers are often used, depending on the characteristic time. Popular examples are: Weissenberg number based on Rouse relaxation time ( $Wi_R$ ) and Weissenberg number based on longest relaxation time ( $Wi_d$ ).

#### Table XIII. Frequently used constants

- $N_{\rm A}$  Avogadro's number,  $6.023 \times 10^{23}$  molecules mol<sup>-1</sup>
- *R* Ideal gas constant, 8.314 J mol<sup>-1</sup> K<sup>-1</sup>
- $k_{\rm B}$  Boltzmann's constant,  $1.38 \times 10^{-23}$  J K<sup>-1</sup>